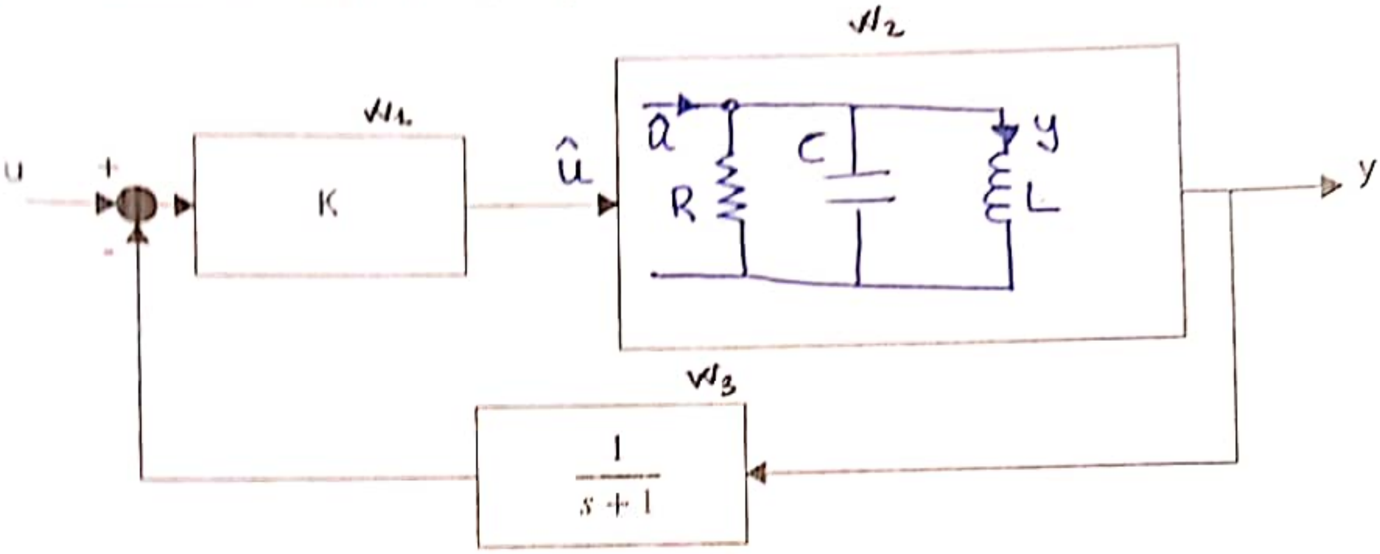
Automatica / Teoria dei Sistemi e Fondamenti di Teoria del Controllo
07/06/2024

Prova A

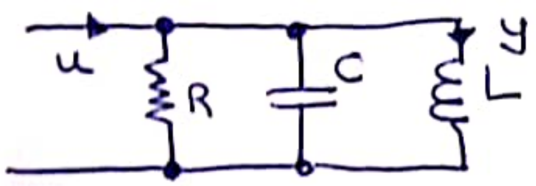
A1. Dato il sistema di figura, determinare una rappresentazione I-S-U e i valori di K per cui il sistema è asintoticamente stabile (R=1, L=1, C=1)



A2. Dato il seguente sistema tracciarne i diagrammi di Bode

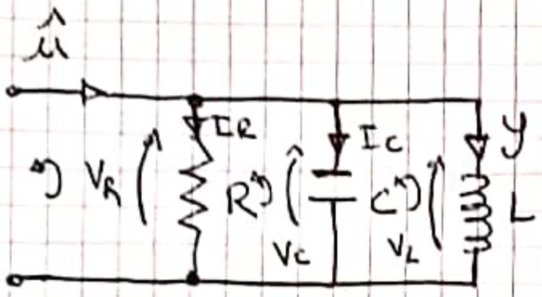
$$W(s) = \frac{-1000\, s^3 + 2 s^2 - 10\, s}{(10^3 s^2 + 2s + 10)(1000\, s^2 + 110\, s + 1)}$$

A3. Tracciare la risposta qualitativa del circuito (R=1,L=1,C=1) ad un forzamento u(t)=-5*1(t-1)



ESERCIZIO A1.

CALCOLO RAPPRESENTAZIONE ISU N_2



$$x = \begin{bmatrix} V_C \\ I_L \end{bmatrix}$$

$$\begin{cases} V_R - V_C = 0 \\ V_C - V_L = 0 \\ \hat{u} - I_R - I_C - I_L = 0 \end{cases}$$

$V_R = V_C = V_L$

$V_R = R I_R$

$I_C = C \dot{V}_C$

$V_L = L \dot{I}_L$

$L \dot{I}_L = V_C$

$I_R = \hat{u} - C\dot{V}_C - I_L$

$V_R = R(\hat{u} - C\dot{V}_C - I_L)$

$V_C = R(\hat{u} - C\dot{V}_C - I_L)$

$$\begin{cases} \dot{V}_C = \dfrac{\hat{u}}{C} - \dfrac{I_L}{C} - \dfrac{V_C}{RC} \\ \dot{I}_L = \dfrac{V_C}{L} \end{cases}$$

$y = I_L$

$$\dot{x}(t) = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & \emptyset \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{C} \\ \emptyset \end{bmatrix} \hat{u}(t)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + \emptyset u$$

$$W_2(s) = C(sI - A)^{-1}B + D$$

$$W_2(s) = [0 \quad 1] \frac{\begin{bmatrix} s & -1 \\ 1 & s+1 \end{bmatrix}}{s^2+s+1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$W_2(s) = \frac{[1 \quad s+1] \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+s+1} = \frac{1}{s^2+s+1}$$

TROVO LA $W_{SERIE} = W_1 \cdot W_2$

$$W_{SERIE} = \frac{K}{s^2+s+1}$$

TROVO LA $W_{RETROAZIONE}$ FRA W_{SERIE} E W_3

$$W_R = \frac{\frac{K}{s^2+s+1}}{1 + \frac{K}{s^2+s+1} \cdot \frac{1}{s+1}} =$$

$$W_R = \frac{\frac{K}{s^2+\cancel{s+1}}}{\frac{(s^2+s+1)(s+1)+K}{(s^2+\cancel{s+1})(s+1)}} = \frac{K(s+1)}{s^3+2s^2+2s+1+K}$$

È UN POLINOMIO DI 3 GRADO

APPLICO ROUTH

$p(\lambda) = s^3 + 2s^2 + 2s + (1+k)$

$$
\begin{array}{c|ccc}
3 & 1 & 2 & 0 \\
2 & 2 & 1+K & 0 \\
1 & a_1 & 0 & \\
0 & 1+K & &
\end{array}
$$

$$a_1 = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1+k \end{vmatrix}}{-2} = \frac{(1+k)-4}{-2}$$

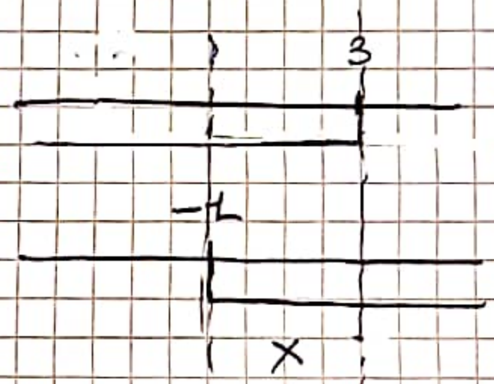
$$a_1 = -\frac{1+k}{2} + 2$$

PER A.S. 1) $a_1 > 0$

2) $K + 1 > 0$

1) $\frac{3}{2} - \frac{k}{2} > 0$ $\quad K < +3$

2 $\quad K + 1 > 0$ $\quad K > -1$



IL SISTEMA E' A.S

PER VALORI DI $K > -1 \vee K < 3$

PER TROVARE LA ISU GLOBALE APPLICO LA FORMULA

$$\frac{k(s+1)}{s^3+2s^2+2s+(1+k)}$$

$$x(t) = \begin{bmatrix} -2 & 1 & 0 \\ -2 & 0 & 1 \\ -(1+k) & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ k \\ k \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x(t) + \emptyset\, u(t)$$

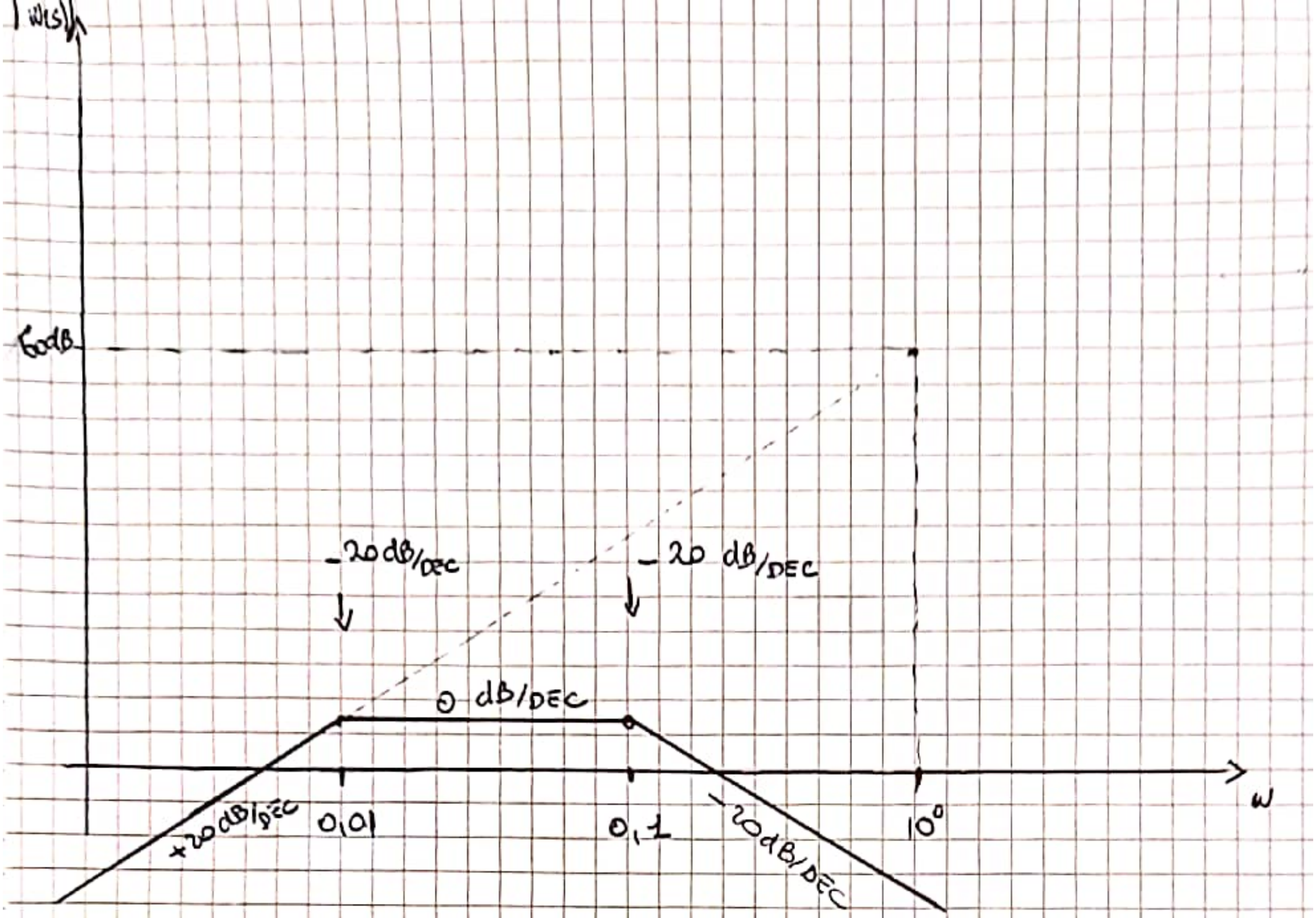
ESERCIZIO A2

$$W(s) = \frac{+s(-1000s^2+2s-10)}{(10^3s^2+2s+10)(1000s^2+110+1)}$$

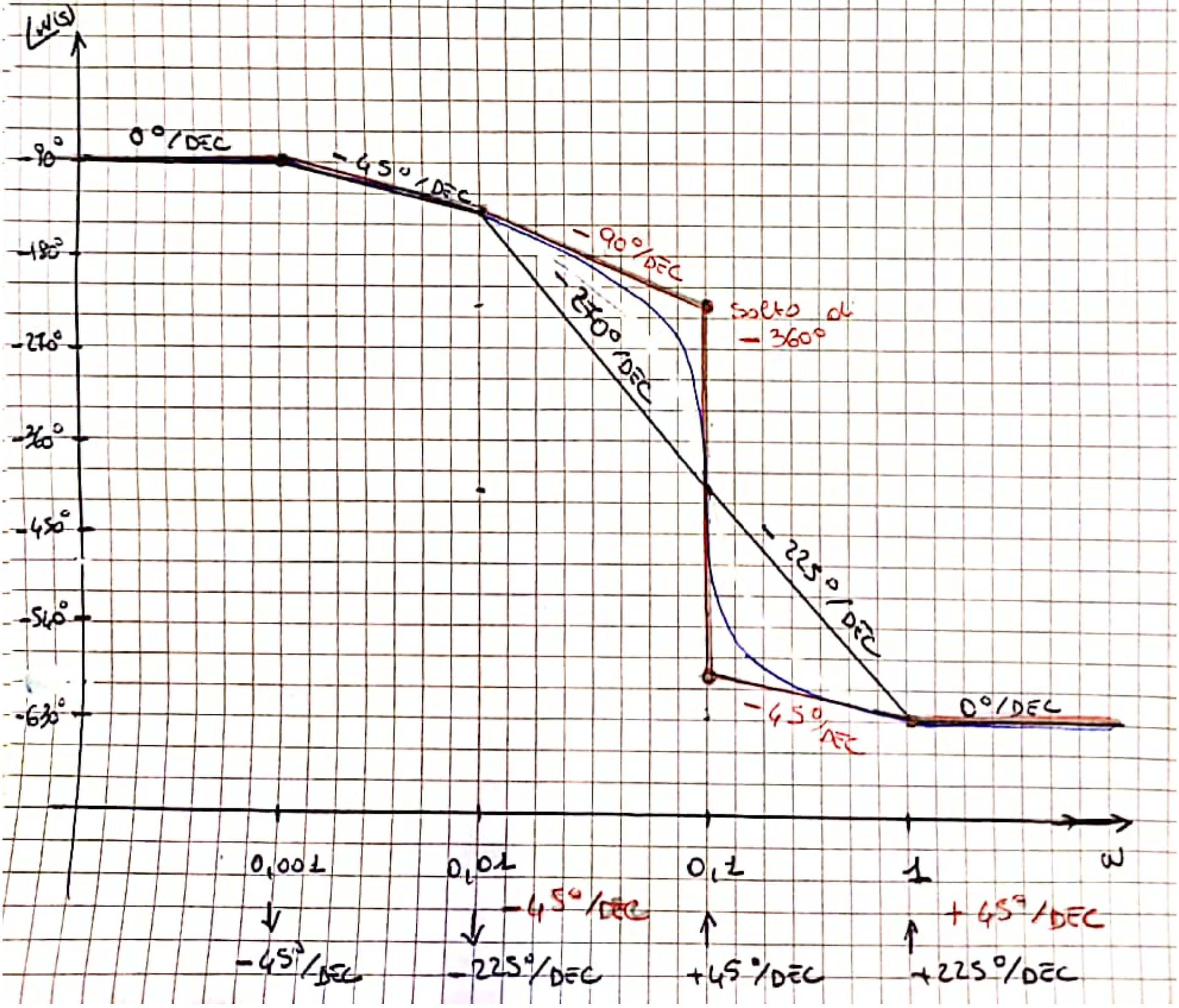
$$W(s) = \frac{-10s\left(\frac{1000}{10}s^2 - \frac{2}{10}s+1\right)}{10\cdot(+0{,}1)(+0{,}01)\left(\frac{10^3}{10}s^2+\frac{2}{10}s+1\right)\left(\frac{s}{0{,}1}+1\right)\left(\frac{s}{0{,}01}+1\right)}$$

$$W(s) = \frac{-1000s\left(\frac{1000}{10}s^2-\frac{2}{10}s+1\right)}{\left(\frac{10^3}{10}s^2+\frac{2}{10}s+1\right)\left(\frac{s}{0{,}1}+1\right)\left(\frac{s}{0{,}01}+1\right)}$$

	MODULO	FASE
$\mu = -1000$	$20\log(\lvert\mu\rvert) = 60\ dB$	$-180°\ \forall\omega$
s	$20\ dB/DEC\ \forall\omega$	$+90°\ \forall\omega$
$\left(\frac{s}{0{,}1}+1\right)^{-1}$	$\omega = 0{,}1$ $-20\ dB/DEC$	$\omega_1 = 0{,}01$ $\omega_2 = 1$ $-45°/DEC$ $+45°/DEC$
$\left(\frac{s}{0{,}01}+1\right)^{-1}$	$\omega = 0{,}01$ $-20\ dB/DEC$	$\omega_1 = 0{,}001$ $\omega_2 = 0{,}1$ $-45°/DEC$ $+45°/DEC$
$\left(\frac{1000}{10}s^2-\frac{2}{10}s+1\right)$	$\omega = 0{,}1$ $+40\ dB/DEC$ $\xi = -0{,}01$	$\omega_1 = 0{,}01$ $\omega_2 = 1$ $-90°/DEC$ $+90°/DEC$
$\left(\frac{1000}{10}s^2+\frac{2}{10}s+1\right)^{-1}$	$\omega = 0{,}1$ $-40\ dB/DEC$ $\xi = 0{,}01$	$\omega_1 = 0{,}01$ $\omega_2 = 1$ $-90°/DEC$ $+90°/DEC$
$\phi_{in} = -90°$	$\phi_{fin} = -630°$	



|W(s)|
60dB
-20dB/DEC
-20 dB/DEC
0 dB/DEC
+20dB/DEC
0,01
0,1
10
-20dB/DEC
ω



∠W(s)
-90°
-180°
-270°
-360°
-450°
-540°
-630°
0°/DEC
-45°/DEC
-90°/DEC
Salto di -360°
-270°/DEC
-225°/DEC
-45°/DEC
0°/DEC
0,001
0,01
0,1
1
ω
-45°/DEC
-45°/DEC
-225°/DEC
+45°/DEC
+45°/DEC
+225°/DEC

ESERCIZIO A.3

$$W(s) = \frac{1}{s^2+s+1}$$

FORZAMENTO:

$$\begin{cases} u(t) = -5 \cdot 1(t-1) \\ \hat{t} = t-1 \rightarrow u(\hat{t}) = -5 \cdot 1(\hat{t}) \\ u(s) = -\frac{5}{s} \end{cases}$$

$$y(0) = \lim_{s\to\infty} \cancel{s} \cdot \frac{1}{s^2+s+1} \cdot \left(-\frac{5}{\cancel{s}}\right) = \emptyset$$

$$\dot{y}(0) = \lim_{s\to\infty} s^2 \cdot \frac{1}{s^2+s+1} \cdot \left(-\frac{5}{\cancel{s}}\right) = \emptyset$$

$$\ddot{y}(0) = \lim_{s\to\infty} s^{\cancel{3}2} \frac{1}{s^2+s+1} \cdot \left(-\frac{5}{\cancel{s}}\right) = -5 < 0$$

$$y_\infty(t) = W(0) \cdot U_0 = -5$$

$$T_{a1} = \frac{4,6}{0,5} = 9,2\,s \quad , \quad N_{os} = \frac{9,2}{7,25} = 1,26$$

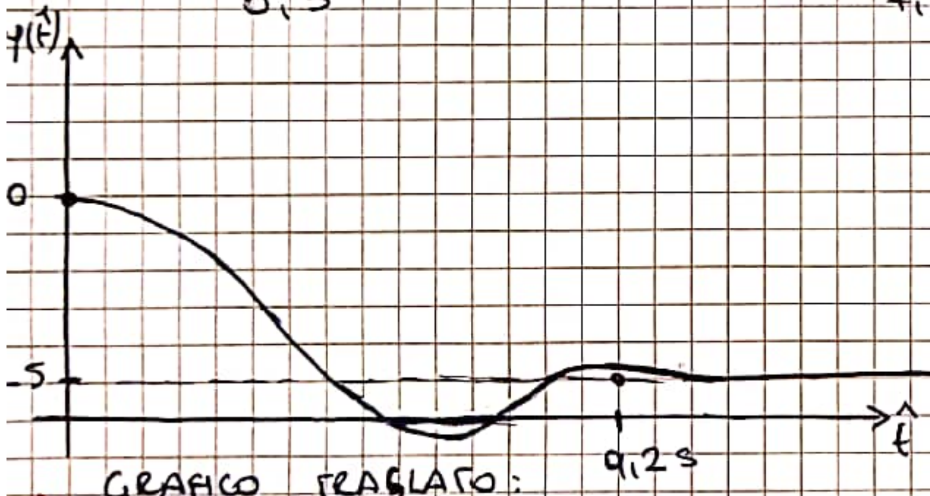


GRAFICO TRASLATO: