Set-theoretic approach for autonomous tracked vehicles involved in post-disaster first relief operations

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Natural disasters: a threat to people's safety

Robotic platforms for rescue and first aid operations

What can we do?

- Implement an effective **emergency response system.**
- Autonomous robotic platforms for first aid and rescue missions after disasters.

Advantages of using robotic systems

- Robots can reach places inaccessible to human operators.
- The use of robots instead of human operators can significantly increase the safety of rescue personnel and the effectiveness of rescue operations.

State of Art

First uses of robots in rescue missions

Some examples are:

- the attacks of September 11 2001 (New York, USA);
- the landslide in La Conchita (U.S.) 2005;
- the Hurricanes Katrina (USA) and Wilma (USA) both in 2005;
- the Midas Gold Mine collapse (U.S.) 2007;
- Fukushima power plant nuclear disaster in 2011 (JPN);

Current status

Nowadays, the average time between the occurrence of a disaster and the actual deployment of a robot is about 6.5 days, much longer than the 48 hours that represent the peak of the threated mortality curve.

Technical issues

- The reasons for this delay depend on many factors:

A) limited autonomy in terms of robot intelligence, power and mobility;

B) insufficient integration with the rescue coordination center;

C) limited capability to coordinate many robotic units deployed in the same operational scenario.

Funding Projects

- *Resilient and Secure Networked Multivehicle Systems in Adversary Environments* granted by the Italian Ministry of University and Research (MUR) within the PRIN 2022 program and European Union - Next Generation EU
- *Cooperative Heterogeneous Multi-drone SYStem for disaster prevention and first response* granted by the Italian Ministry of University and Research (MUR) within the PRIN 2022 PNRR program, funded by the European Union through the PNRR program

Objectives

- We address some of the various technical and technological issues that currently limit the use of robotic systems in disaster relief in order to provide a proof of concept for the entire (multi)-robotic system .

Proposal keypoints

Problem specifically addressed

Feasible motion planning problem for an autonomous mobile robot moving from the starting point to the target position in a cluttered scenario.

Application aspect

Main claim: the proposed strategy is able to explicitly account for model uncertainties and constraints which alter in non-negligible manner the robot's motion capabilities.

Theoretical aspect

From a theoretical point of view, this is achieved by resorting to the concept of one-step-ahead controllable set.

Mathematical preliminaries

- Assuming that $t_k = k \cdot T_c$, where T_c is the sampling time, we consider the following state space description of the discrete-time quasi-LPV model of robot

$$x(t_{k+1}) = A(\alpha(t_k)) \cdot x(t_k) + B(\alpha(t_k)) \cdot u(t_k) \tag{1}$$

- Assuming that the matrices $A(\alpha(t))$ and $B(\alpha(t))$ depend affinely on $\alpha(t)$, the following polytopic representation is considered

$$\{A(\alpha(t_k)), B(\alpha(t_k))\} \in Co\{(A_s, B_s)\} \tag{2}$$

where $A_s = A(\alpha_s)$ and $B_s = B(\alpha_s)$ with

$$Co\{(A_s, B_s)\} = \sum_{s=1}^{M} \pi_s(A_s, B_s). \tag{3}$$

One-step ahead ellipsoidal controllable sets

- Consider the uncertain model (1) and assume that the control input is subject to the following saturation:

$$u(t) \in \mathcal{U},\ \forall t \geq 0,\ \mathcal{U} := \{u \in \mathbb{R}^m : u^T u \leq \overline{u}\} \tag{4}$$

  with $\overline{u} > 0$.
- Let's suppose that the following constraint holds:

$$x(t) \in \mathcal{X},\ \forall t \geq 0,\ \mathcal{X} := \{x \in \mathbb{R}^n : x^T P_q x \leq 1,\ P_q > 0\} \tag{5}$$

- To control the robot's pose to zero, suppose we use the following control law:

$$u(t) = K \cdot q(t) \tag{6}$$

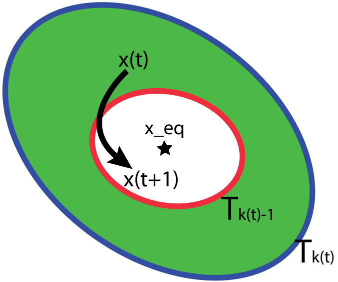
- $\mathcal{T}_0$ is a positively invariant ellipsoidal set.

One-step ahead controllable sets

The ellipsoidal sets of states *i*-steps controllable to $\mathcal{T}$

$$\mathcal{T}_0 := \mathcal{T}$$

$$\mathcal{T}_j := \{x \in \mathcal{X} : \exists\, u \in \mathcal{U} : A_i x + B_i u \in \mathcal{T}_{j-1}\}$$

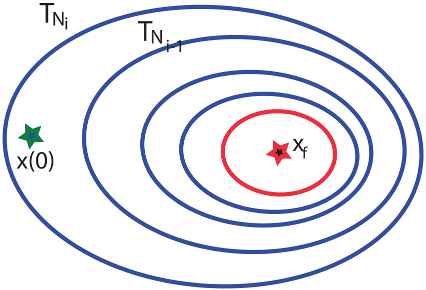


$$\mathcal{U} := \{u \in \mathbb{R}^m : u^T u \leq \overline{u}\} \tag{7}$$

$$\mathcal{X} := \{x \in \mathbb{R}^n : x^T P_q x \leq 1,\ P_q > 0\} \tag{8}$$

$\mathcal{T}_j$ is the largest ellipsoidal set that is compatible with the constraint (8) and includes $\mathcal{T}_{j-1}$, so that each $x(t)$ belonging to $\mathcal{T}_j$ can be controlled to $\mathcal{T}_{j-1}$ in one step by using a control action compatible with the constraint (7).

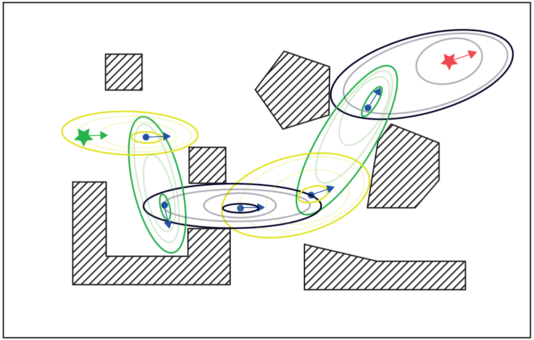
One-step ahead controllable sets Procedure



- Let $x(0)$ and x_f be the initial and goal positions respectively.
- Assume that there is a sequence of N_i one-step ahead controllable set.
- There is a feasible path from $x(0)$ to x_f compatible with all prescribed constraints, allowed uncertainties and obstacles in the cluttered scenario.
- $x(0)$ is controllable to x_f in at least N_i moves .

Motion planning algorithm: Offline Phase

- Repeat the process until the starting point is reached from the target position setting some intermediate waypoints if necessary.



- It is possible to guarantee that a path is feasible given the uncertain dynamics of robot, constraints and obstacles in the cluttered scenario.

Motion planning algorithm: Online Phase

Let $\mathcal{T}_j$ be the smallest one-step controllable set containing $x(t)$:

- If $j = 0$, the control action is computed according to the following formula: $u(t) = K \cdot x(t)$: $x(t)$ is in $\mathcal{T}_0$, the control action is compatible with all the prescribed constraints.
- If $j \neq 0$, the control action can be computed by solving the following minimization problem:

$$\min_{u \in \mathcal{U}} \|A_s \cdot x(t) + B_s \cdot u\|^2_{\mathcal{T}_{j-1}}, \; s = 1 \cdots N_i \tag{9}$$

s.t.

$$A_s \cdot x(t) + B_s \cdot u \in \mathcal{T}_{j-1} \tag{10}$$

Remark

The control action $u(t) \in \mathcal{U}$ fulfils the prescribed constraints such that the state evolution $A(\alpha(t))x(t) + B(\alpha(t))u$ belongs to $\mathcal{T}_{j-1}$

Example

Robot characteristics

A tracked mobile robot with $R = 0.07$ [m], $D = 0.5$ [m], and with the following forward and rotational velocity constraints were considered:

a) $V \in [0, 0.4]$ [m/s];

b) $\omega \in [-120, 120]$ [deg/s]

Uncertainties (μ_r, μ_l)

It is assumed that the sliding coefficients for the two tracks can take values between [0.85, 1.15].

$$\dot{q}_L(t) = R_E^L(\theta_0) \cdot G(t) \cdot J \cdot H(t) \cdot J^{-1} \cdot u(t) \tag{11}$$

$$R_E^L(\theta_0) = \begin{bmatrix} cos(\theta_0) & sin(\theta_0) & 0 \\ -sin(\theta_0) & cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$H(t) = \begin{bmatrix} \mu_r(t) & 0 \\ 0 & \mu_l(t) \end{bmatrix} \tag{13}$$

$$J = \begin{bmatrix} R/2 & R/2 \\ R/D & -R/D \end{bmatrix} \tag{14}$$

$$G(t) = \begin{bmatrix} cos(\theta(t)) & 0 \\ sin(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Example

Using Euler's approximation with sampling time T_s, the following discrete-time quasi-LPV model state space description comes out

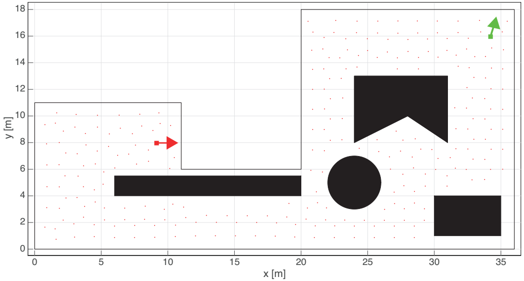
$$q_L(t_{k+1}) = A(\alpha(t_k)) \cdot q_L(t_k) + B(\alpha(t_k)) \cdot u(t_k) \tag{16}$$

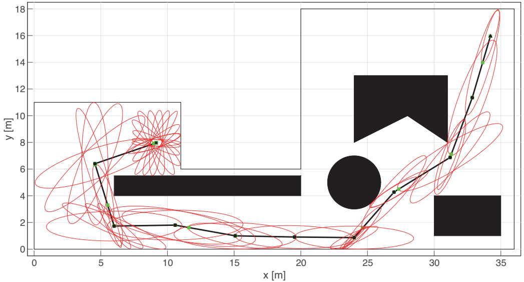
$\{A(\alpha(t_k)), B(\alpha(t_k))\} \in Co\{(A_s, B_s)\}$ being $Co\{(A_s, B_s)\} = \sum_{s=1}^{N}(A_s, B_s)$

Mathematical Model

- $t_k = k \cdot T_s$ with $T_s = 0.4[s]$
- Discrete uncertain linear model having $N = 18$ vertices .
- A robust control action $u(t_k) = K \cdot q_L(t_k)$ guarantees the stability of (16) in the ellipsoidal set T_0.
- A family of 41 one-step ahead controllable sets were then defined

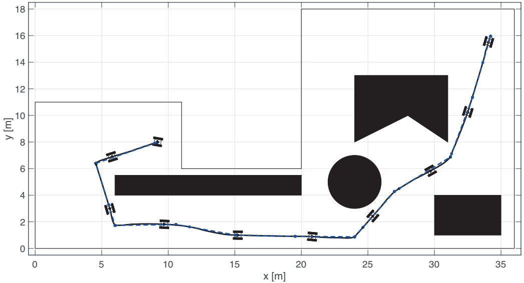
Example - Offline Phase

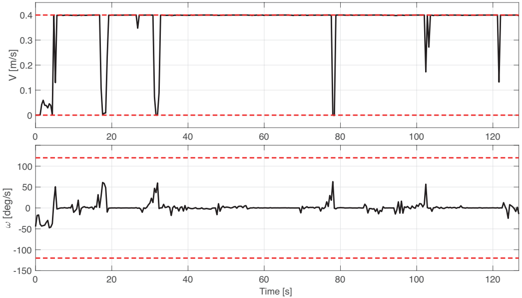




- A path involving a sequence of 10 segments with a length of 46.861 [m] was defined (black lines)
- To guarantee the feasibility of selected path, 72 sequences of ellipsoidal sets were identified (in red the projections onto the $\{x, y\}$-plane of the ellipsoidal regions appropriately rotated)

Example - Online Phase



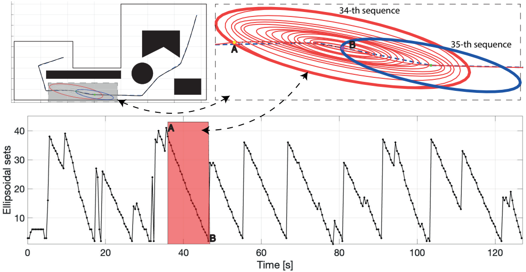


The control action is calculated according to the proposed online phase by solving an optimization problem whose feasibility is guaranteed by the calculations performed in the offline phase.

Example - How does the online phase work?

At position **A**, the robot belongs to sequence 34-th.

The control action is calculated to reach the point **B** which allows the robot to traverse the ellipsoidal sets of the 34-th sequence (*from the set with index* 41 *to the ellipsoidal set with index* 2) .



- When the robot reaches the ellipsoidal set 2, the robot pose is contained in the ellipsoidal controllable set with index 29 of sequence 35-th. This enables the transition to the next sequence.
- Similar behavior can be observed all along the path. This is the key procedure by which the robot reaches the prescribed final pose.
- Each jump upwards represents a transition to the successive sequence of ellipsoidal sets.

Principal Keypoints

- An algorithm for computing an admissible motion sequence for an autonomous mobile robot moving in a cluttered environment.
- The proposed algorithm ensures collision-free motion compatible with model uncertainties and constraints.
- Set-theoretic argument : one-step-ahead controllable set.
- In the Offline phase, the sequences of one-step-ahead controllable sets are planned. An admissible collision-free motion strategy is defined to reach the prescribed target accounting for model uncertainties, constraints and obstacles in the cluttered environment.
- Online phase. Calculation of the robot control action. The control action is computed in terms of online solution of a constrained minimization problem whose feasibility is ensured by the analysis performed in the offline phase.
- In order to show the effectiveness of the proposed algorithm, a numerical example has been provided.

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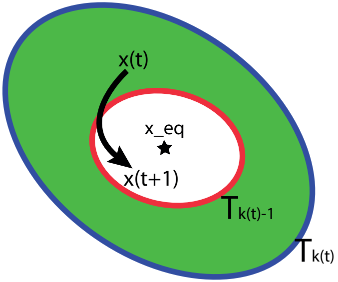

$$\mathcal{U} := \{u \in \mathbb{R}^m : u^T u \leq \overline{u}\}$$

$$\mathcal{X} := \{x \in \mathbb{R}^n : x^T P_q x \leq 1,\ P_q > 0\}$$

. Ellipsoidal set $\mathcal{T}_0$ is positively invariant for discrete-time uncertain system: there exists a control law $u(t) \in \mathcal{U}$ such that once the closed-loop solution $x_{CL}(t)$ enters inside that set at any given time t_0, it remains in it for all future instants, i.e. $x_{CL}(t_0) \in \mathcal{T}_0 \rightarrow x_{CL}(t) \in \mathcal{T}_0, \forall t \geq t_0$.