A model predictive control architecture for autonomous vehicles moving in uncertain scenarios

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Robotic platforms for rescue and first aid operations








Robotic platforms for rescue and first aid operations

Preliminary considerations

- Implement an effective **emergency response system.**
- Autonomous robotic platforms for first aid and rescue missions after disasters.

Advantages of using robotic systems

- Robots can reach places inaccessible to human operators.
- The use of robots instead of human operators can significantly increase the safety of rescue personnel and the effectiveness of rescue operations.

State of Art

First uses of robots in rescue missions

Some examples are:

- the attacks of September 11 2001 (New York, USA);
- the landslide in La Conchita (U.S.) 2005;
- the Hurricanes Katrina (USA) and Wilma (USA) both in 2005;
- the Midas Gold Mine collapse (U.S.) 2007;
- Fukushima power plant nuclear disaster in 2011 (JPN);

Current status

Nowadays, the average delay between the occurrence of a disaster and the deployment of robots is about 6.5 days - significantly longer than the 48 hours that represent the critical peak of the mortality risk curve.

Technical issues

- The reasons for this delay depend on many factors:

A) limited autonomy in terms of robot intelligence, power and mobility;

B) insufficient integration with the rescue coordination center;

C) limited capability to coordinate many robotic units deployed in the same operational scenario.

Funding Projects

- *Resilient and Secure Networked Multivehicle Systems in Adversary Environments* granted by the Italian Ministry of University and Research (MUR) within the PRIN 2022 program and European Union - Next Generation EU
- *Cooperative Heterogeneous Multi-drone SYStem for disaster prevention and first response* granted by the Italian Ministry of University and Research (MUR) within the PRIN 2022 PNRR program, funded by the European Union through the PNRR program

Objectives

- We address some of the various technical and technological issues that currently limit the timely deployment of robotic platforms in disaster relief.

Proposal keypoints

- The core of this paper relies on the design of an original networked control architecture for efficiently driving autonomous ground vehicles in rescue missions within cluttered, static and unknown environments.

- From a theoretical point of view, this is accomplished by resorting to a set-theoretic receding horizon control that includes the following:

    1. a protocol capable of adequately exploiting past data without compromising feasibility, closed-loop stability, and obstacle avoidance properties;
    2. the development of a switch-like control strategy responsible for modifying the trajectory tube whenever necessary to avoid any collision risk.

- Application aspect: path planning and control unit are placed on the remote side of control system to reduce on-board energy consumption caused by computational tasks.

Mathematical preliminaries

- Assuming that $t_k = k \cdot T_c$, where T_c is the sampling time, we consider the following state space description of the discrete-time quasi-LPV model of robot

$$x(t_{k+1}) = A(\alpha(t_k)) \cdot x(t_k) + B(\alpha(t_k)) \cdot u(t_k) \tag{1}$$

- Assuming that the matrices $A(\alpha(t))$ and $B(\alpha(t))$ depend affinely on $\alpha(t)$, the following polytopic representation is considered

$$\{A(\alpha(t_k)), B(\alpha(t_k))\} \in Co\{(A_s, B_s)\} \tag{2}$$

where $A_s = A(\alpha_s)$ and $B_s = B(\alpha_s)$ with

$$Co\{(A_s, B_s)\} = \sum_{s=1}^{M} \pi_s (A_s, B_s). \tag{3}$$

Robust One-Step Ahead Controllable Set (ROSAC)

- Consider the uncertain model (1) and assume that the following ellipsoidal constraints:

$$u(t) \in \mathcal{U}, \forall t \geq 0, \mathcal{U} := \{u \in \mathbb{R}^m : u^T u \leq u_{max}^2\} \tag{4}$$

$$x(t) \in \mathcal{X}, \forall t \geq 0, \mathcal{X} := \{x \in \mathbb{R}^n : x^T x \leq x_{max}^2\} \tag{5}$$

- To control the robot's pose, suppose we use the following Robust Control Law (RCL) which guarantees the robust stability of (1) according to prescribed constraints

$$u(t) = K \cdot x(t - \tau(t)), \tau(t) \leq \bar{\tau}, \forall t \geq 0 \tag{6}$$

within the Robust Positively Invariant (RPI) ellipsoidal set $\mathcal{T}_0$, $\bar{\tau}$ represents an upper bound of the time delay introduced in the control loop.

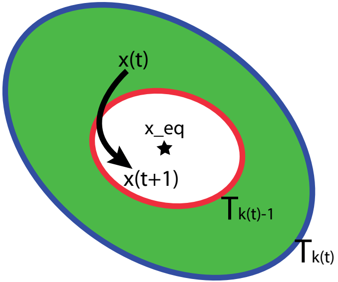
Robust Positively Invariant set : if the system's state starts within RPI, it will remain within RPI for all future time steps, fulfilling all prescribed constraints, given the uncertain dynamics and allowable time delay.

Robust One-Step Ahead Controllable Set (ROSAC)

The ellipsoidal sets of states *i*-steps controllable to $\mathcal{T}$

$$\mathcal{T}_0 := \mathcal{T}$$

$$\mathcal{T}_j := \{x \in \mathcal{X} : \exists\, u \in \mathcal{U} : A_i x + B_i u \in \mathcal{T}_{j-1}\}$$

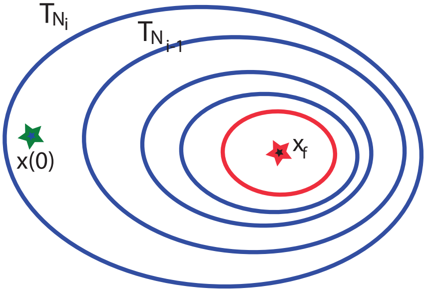


$$u(t) \in \mathcal{U},\ \forall t \geq 0,\ \mathcal{U} := \{u \in \mathbb{R}^m : u^T u \leq u_{max}^2\} \quad (7)$$

$$x(t) \in \mathcal{X},\ \forall t \geq 0,\ \mathcal{X} := \{x \in \mathbb{R}^n : x^T x \leq x_{max}^2\} \quad (8)$$

$\mathcal{T}_j$ is the largest ellipsoidal set that is compatible with the constraint (8) and includes $\mathcal{T}_{j-1}$, so that each $x(t)$ belonging to $\mathcal{T}_j$ can be controlled to $\mathcal{T}_{j-1}$ in one step by using a control action compatible with the constraint (7). $\mathcal{T}_j$ is computed by solving SDP optimization problem involving LMIs constraints.

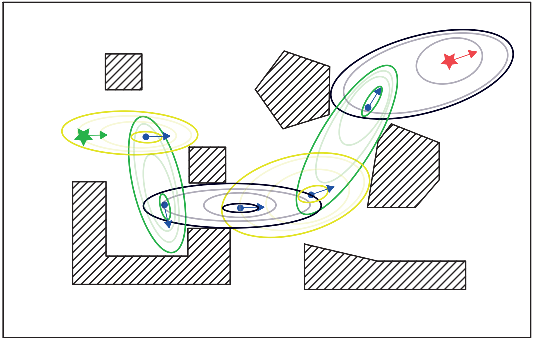
ROSAC Procedure



- Assume that there is a sequence of N_i one-step ahead controllable set.
- There is a feasible path from $x(0)$ to x_f compatible with all prescribed constraints, allowed uncertainties and obstacles in the cluttered scenario.
- $x(0)$ is controllable to x_f in at least N_i moves .

Motion planning algorithm: Offline Phase

- Repeat the process until the starting point is reached from the target position setting some intermediate waypoints if necessary.



- It is possible to guarantee that a path is feasible given the uncertain dynamics of robot, constraints and obstacles in the cluttered scenario.

Motion planning algorithm: Online Phase

Let $\mathcal{T}_j$ be the smallest one-step controllable set containing $x(t)$:

- If $j = 0$, the control action is computed according to RCL guaranteeing all the prescribed constraints.
- If $j \neq 0$, the control action can be computed by solving the following minimization problem:

$$\min_{u \in \mathcal{U}} \|A_s \cdot x(t) + B_s \cdot u\|^2_{\mathcal{T}_{j-1}},\ s = 1 \cdots N_i \quad (9)$$

s.t.

$$A_s \cdot x(t) + B_s \cdot u \in \mathcal{T}_{j-1} \quad (10)$$

Remark

The control action $u(t) \in \mathcal{U}$ fulfils the prescribed constraints for any allowable time delay, such that the state evolution $A(\alpha(t))x(t) + B(\alpha(t))u$ belongs to $\mathcal{T}_{j-1}$

Example

Jaguar V4 by Dr.Robot

It is a tracked robotic platform with a footprint of $70 \times 50\ cm^2$. Forward and rotational speeds satisfy the following bounds:

a) $V \in [0, 0.4]$ [m/s];

b) $\omega \in [-60, 60]$ [deg/s]

Uncertainties (μ_r, μ_l)

It is assumed that the sliding coefficients for the two tracks can take values between [0.8, 1.2].

$$\dot{q}_L(t) = R_E^L(\theta_0) \cdot G(t) \cdot J \cdot H(t) \cdot J^{-1} \cdot u(t) \tag{11}$$

$$R_E^L(\theta_0) = \begin{bmatrix} cos(\theta_0) & sin(\theta_0) & 0 \\ -sin(\theta_0) & cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$H(t) = \begin{bmatrix} \mu_r(t) & 0 \\ 0 & \mu_l(t) \end{bmatrix} \tag{13}$$

$$J = \begin{bmatrix} 1/2 & 1/2 \\ 1/D & -1/D \end{bmatrix} \tag{14}$$

$$G(t) = \begin{bmatrix} cos(\theta(t)) & 0 \\ sin(\theta(t)) & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Example

Using Euler's approximation with sampling time T_s, the following discrete-time quasi-LPV model state space description comes out

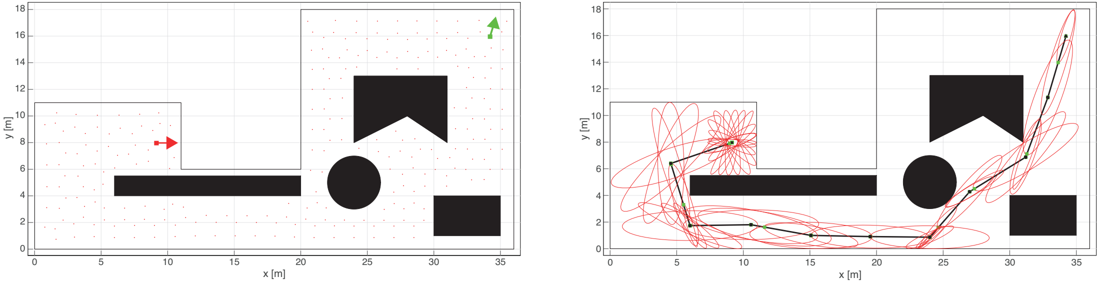
$$q_L(t_{k+1}) = A(\alpha(t_k)) \cdot q_L(t_k) + B(\alpha(t_k)) \cdot u(t_k) \tag{16}$$

$\{A(\alpha(t_k)), B(\alpha(t_k))\} \in Co\{(A_s, B_s)\}$ being $Co\{(A_s, B_s)\} = \sum_{s=1}^{M}(A_s, B_s)$

Mathematical Model

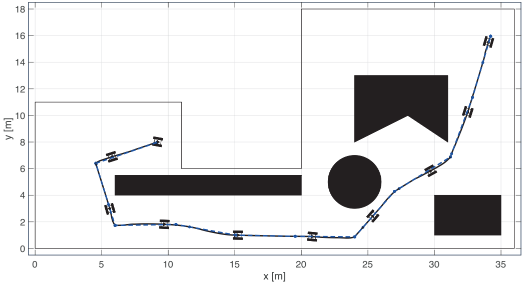
- $t_k = k \cdot T_s$ with $T_s = 0.4$[s]
- Discrete uncertain linear model having $M = 12$ vertices .
- A robust control action $u(t_k) = K \cdot q_L(t_k - \tau(t_k))$ guarantees the stability of (16) in RPI ellipsoidal set T_0 being $\tau(t_k) \leq \bar{\tau}$, $\bar{\tau} = 2$

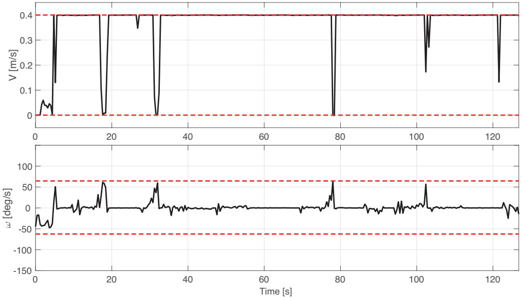
Example- Known Scenario- Offline Phase



- A path involving a sequence of 10 segments with a length of 46.861 [m] was defined (black lines)
- To guarantee the feasibility of selected path, 72 sequences of ellipsoidal sets were identified (in red the projections onto the $\{x, y\}$-plane of the ellipsoidal regions appropriately rotated)

Example - Online Phase



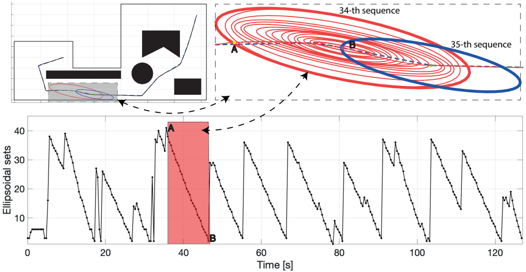


The control action is calculated according to the proposed online phase by solving an optimization problem whose feasibility is guaranteed by the calculations performed in the offline phase.

Example - How does the online phase work?

At position **A**, the robot belongs to sequence 34-th.

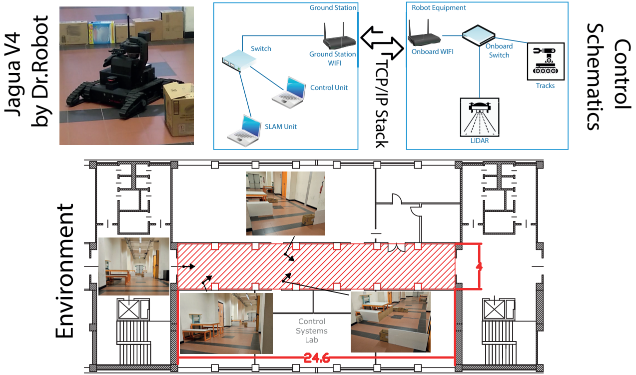
The control action is calculated to reach the point **B** which allows the robot to traverse the ellipsoidal sets of the 34-th sequence (*from the set with index* 41 *to the ellipsoidal set with index* 2) .



- When the robot reaches the ellipsoidal set 2, the robot pose is contained in the ellipsoidal controllable set with index 29 of sequence 35-th. This enables the transition to the next sequence.
- Similar behavior can be observed all along the path. This is the key procedure by which the robot reaches the prescribed final pose.
- Each jump upwards represents a transition to the successive sequence of ellipsoidal sets.

Experimental Setup - Partially known Scenario

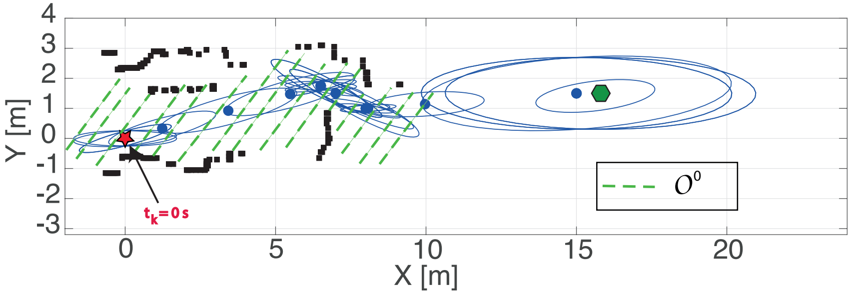
The proposed algorithm was used for the following experimental test.



Robot is equipped with a front-mounted LIDAR sensor providing a maximum depth perception up to $R = 10\ m$. Measurement campaign has estimated Maximum Allowable Transfer Interval ($\bar{\tau} = 2$) and sliding coefficients. SLAM algorithm is used for navigation purposes.

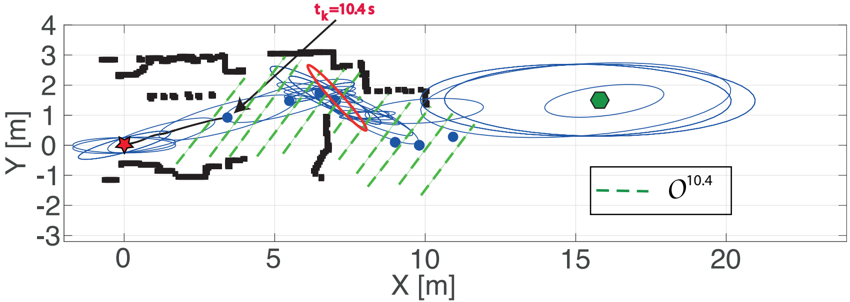
Experiment

- By resorting to the available information on the environment at the time instant $t_k = 0$, the **Path Planner** has determined a sequence Γ of twenty way-points connecting starting (red star) and ending poses (green hexagon).
- For all possible occurrences of the induced delay the relative family of ROSAC sequences, is computed. Each ROSAC consists of $N = 41$ ellipsoids.

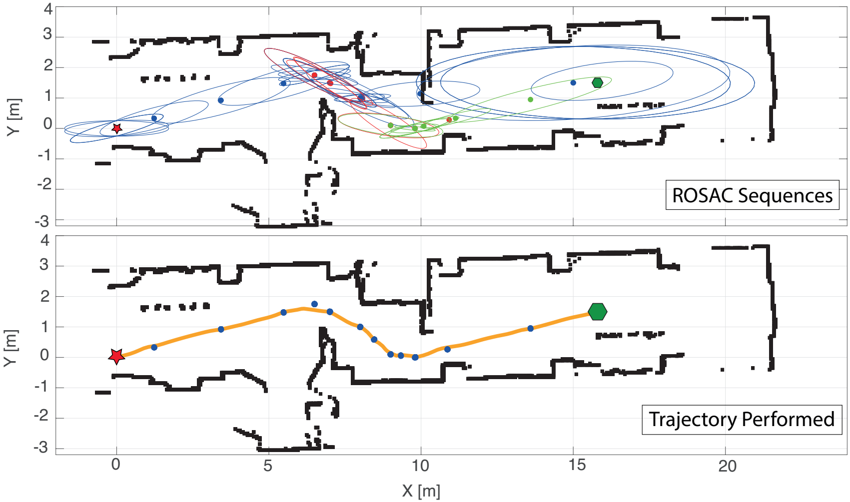


Experiment

- At $t_k = 10.4\,\text{s}$, the vehicle has moved where a non-viability condition occurs : the state trajectory tube resulting from the nominal path Γ is no longer viable due to a collision with a previously unknown obstacle.
- On the basis of updated information of the environment: a new way-points are determined and the overlapped ellipsoids are computed.
- During the update, the vehicle is driven through the previous trajectory tube up to the last safe ROSAC sequence.



Experiment



ROSAC Sequences
Trajectory Performed
Y [m]
Y [m]
X [m]
4
3
2
1
0
-1
-2
-3
0
5
10
15
20

Principal Keypoints

- Set-theoretic argument : one-step-ahead controllable set.
- A networked set-theoretic based model predictive control architecture has been developed for autonomous robots operating in cluttered, static and unknown environments.
- It has been formally shown that the proposed algorithm is capable to guarantee the constraint satisfaction and anti-collision capabilities despite of time-delay occurrences along the communication channel.
- Real experiments have shown the effectiveness of this solution when unpredictable obstacles prevent the use of the nominal controller.
- Future studies will attempt to extend the approach to multi-robot configurations, as this may improve the chances of mission success in complex operations.

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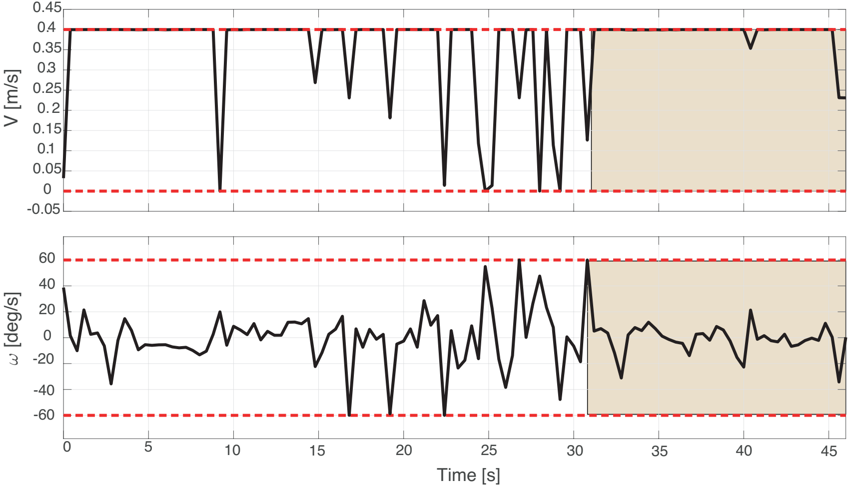
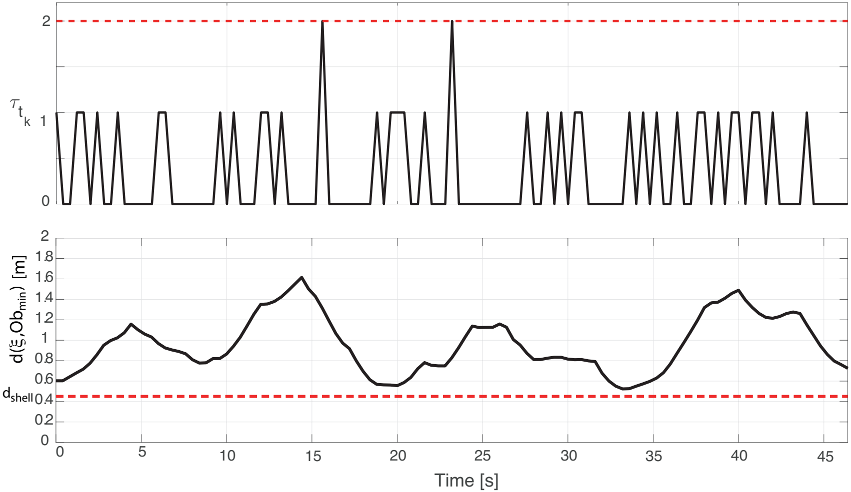
Experiment



0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
V [m/s]
60
40
20
0
-20
-40
-60
ω [deg/s]
0
5
10
15
20
25
30
35
40
45
Time [s]

Experiment



2
τ_{t_k} 1
0
2
1.8
1.6
1.4
1.2
1
0.8
0.6
d_{shell} 0.4
0.2
0
$d(\xi, Ob_{min})$ [m]
0
5
10
15
20
25
30
35
40
45
Time [s]